Electronic Supplementary Material for

A model balancing cooperation and competition can explain our right-handed world and the dominance of left-handed athletes

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S1. COMPARISON TO PREVIOUS MODELS

Our model is not the first to account for population-level handedness in human populations. Several previous models explain this outcome as an evolutionarily stable strategy due to frequency dependent fitness functions (e.g., [10],[14], [17], [18]). For example, in [17], Billiard et al. combine a constant fitness cost for left-handers with a frequency dependent fitness advantage for individuals in the minority. Similarly, in the model of Ghirlanda et al., members of the minority experience an advantage in antagonistic interactions and a disadvantage in synergistic interactions. Their fitness is defined in terms of a balance between these two components. In both models, equilibrium is obtained when the fitness of left- and right-handed individuals are equal [10].

In this paper, however, we employ a different approach. We argue that segments of a population will switch handedness over long time scales according at a rate determined by a probabilistic function. Thus equilibrium is obtained when the overall transition rates between left- and right-handed groups balance. These probabilistic transition rates should be related to the comparative fitness of the members of the groups.

We now determine the relationship between fitness functions \( f_L(l) \), \( f_R(l) \) and probabilistic transition rates \( P_{RL}(l) \). Recall that \( P_{RL}(l) \) represents the probabilistic transition rate from right to left. Clearly, \( P_{RL}(l) \) must be non-negative. Also, it should reach a maximum (minimum) when the difference in fitness between left- and right-handers is at a maximum (minimum). The simplest example of such a function is \( P_{RL}(l) = A(f_L(l) - f_R(l)) + B \), where \( A,B > 0 \) are constants guaranteeing that \( P_{RL}(l) \) remains positive. By symmetry, \( f_L(l) = f_R(1-l) \) and \( P_{RL}(l) = P_{LR}(1-l) \). Thus \( P_{LR}(l) = A(f_R(l) - f_L(l)) + B \). Transition rates defined in this way satisfy the symmetry relation \( P_{RL}(l) + P_{RL}(1-l) = 2P_{RL}(1/2) \) and, as such, are sigmoidal for many different types of fitness functions.

We should note that in this model, the probability \( P_{RL}(l)\Delta t \) that a given individual switches from right to left within time \( \Delta t \) is non-zero even when \( f_R(l) > f_L(l) \). In such a situation, the probability that a given left-hander switches is \( P_{LR}(l)\Delta t > P_{RL}(l)\Delta t \). However, if right-handers are much more prevalent than left-handers, the total number of switches from right to left may still outweigh the number from left to right. In other words, it is possible that \( P_{LR}(l)\Delta t < rP_{RL}(l)\Delta t \). This would cause the fraction left-handed to increase despite right-handers having a higher fitness.

Also, if for some \( l_0 \), \( f_R(l_0) = f_L(l_0) \), then \( P_{RL}(l_0) = P_{LR}(l_0) = B \) and \( \frac{dP}{dl}\bigg|_{l_0} = (1 - l_0)B - l_0(B) = (1 - 2l_0)B \). So if \( l_0 < 1/2 \), the fraction left-handed will increase even when the fitness is equal for left- and right-handers. In our model, having equal fitnesses does not necessarily lead to equilibrium.

Despite the differences between our formulation and a fitness-based formulation, the predictions are similar. If we use the fitness functions \( f_L(l) = (1 - c)e^{-k_1l} + c(1 - e^{-k_2l}) \) similar to those proposed by Ghirlanda et al. we can generate \( P_{RL}(l) \) and \( P_{LR}(l) \) (which are sigmoid as expected). In figure S1 we plot the resulting function \( \frac{dP}{dl} \) for appropriate parameter values (such as \( A = 0.0144, B = 0.0047, c = 0.6964, k_u = 3.7745, k_s = 1.9974 \)) and observe a graph very similar to figure S2.

Despite the similarities between the predictions of equation (2.2) and those of previous models, there are various advantages to the novel approach introduced in this paper. The most significant feature of equation (2.2) is its generality. For particular choices of the transition rates, it can reproduce the results of more specialized models. However, it can also describe much richer dynamics. Thus it could easily be applied to selection for other traits that involve frequency dependent competition within and between two subpopulations.

Despite its flexibility, this model is also very robust. Given weak restrictions on the slope and curvature of the transition rates, the qualitative dynamics are insensitive to the particular functional form selected. For example, the equilibrium curves displayed in figure 1 of the main text are qualitatively similar to those observed for a variety of different transition rates including sigmoid functions, exponentials and power laws.

Equation (2.2) is also continuous and dynamic. It can be used to predict not only equilibrium solutions, but also the transient population dynamics. It uses intuitive quantities that, at least in theory, could be determined empirically.
Figure S1. Comparison of probabilistic and fitness-function models. Solid blue line: the function $\frac{dl}{dt}$ [%/yr] generated using generic sigmoid $P_{RL}$ in the probabilistic model from equation (2.2) of the main text. Dashed red line: the function $\frac{dl}{dt}$ [%/yr] generated using $P_{RL}$ implied by fitness functions proposed by Ghirlanda et al. [10].

Given sufficient data on the inheritance of handedness over generations, the probabilistic transition rate could be calculated for particular values of $l$. Fitness functions, on the other hand, are abstract quantities that, though useful from a theoretical perspective, are not directly measurable in the real world. In light of these advantages, our model could be considered a generalization of and improvement upon previous fitness-based models for population-level handedness.

S2. PHENOTYPIC MODEL FOR POPULATION DYNAMICS

The probabilistic model described in the main text provides a description of the population dynamics from a top-down perspective. However, it may be more intuitive to consider the dynamics from a bottom-up perspective. Here we develop a model using reproductive fitness arguments on the level of individuals and show that this produces essentially the same result.

Iterative Model

Let us define $N$ to be the total population size, and $L$ and $R$ to be the number of left- and right-handers, respectively. We make the simplifying assumption that $L + R = N$, i.e., there are no ambidextrous individuals. Also, define handedness fractions $r = R/N$ and $l = L/N$ so that $r + l = 1$.

Suppose that in this population, individuals repeatedly pair off and reproduce, adding new individuals to the population in each generation. From an evolutionary perspective, the expected number of offspring that individuals produce should be dependent on their fitness. With all other factors being equal, an individual’s fitness should be determined by his or her handedness and the distribution of handedness in the population. Thus we define $b_R(l)$ and $b_L(l)$ to be the expected number of offspring born to right- and left-handers.

It is well-established that there is a genetic component to handedness [7]. Thus the handedness of offspring will be related to the handedness of the parents. We define $\sigma_{XY}$ as the probability that a pairing $XY$ produces left-handed offspring, where $X$ and $Y$ represent the dominant hands of the parents ($1 - \sigma_{XY}$ corresponds to the probability of right-handed offspring). We expect $\sigma_{XY} = \sigma_{XY}(l)$ to be a frequency dependent function. This assumption can be motivated by a simple example.

Suppose that left-handedness is a recessive trait. Under this assumption, the probability that a right-hander carries a recessive “lefty” allele will be dependent on the fraction left-handed within the population, and as such, the probability that a right-hander bears left-handed off-spring will be frequency dependent. In all likelihood, the genetic mechanism for handedness is more complex, but the same principle will hold. That is, the distribution of handedness in the population will reflect the distribution of handedness in the gene pool, and consequently the probability of
inheriting a given phenotype will be dependent on genotypes of the parents and indirectly on the fraction left-handed within the population as a whole.

There are 6 possible reproductive interactions (we ignore gender effects here for simplicity—including them should not qualitatively change the results):

\[ R + R \frac{\sigma_{RR}}{\Delta t} L \quad R + L \frac{\sigma_{RL}}{\Delta t} L \quad L + L \frac{\sigma_{LL}}{\Delta t} L \]

\[ R + R \frac{1 - \sigma_{RR}}{\Delta t} R \quad R + L \frac{1 - \sigma_{RL}}{\Delta t} R \quad L + L \frac{1 - \sigma_{LL}}{\Delta t} R . \]

At each iteration, we suppose that a fraction \( D \) of current members dies off. Then the number of left- and right-handers in the new generation is:

\[ L_{n+1} = \left[ \frac{b_R \sigma_{RR} R^2_n + (b_R + b_L) \sigma_{RL} R \sigma_{LL} L^2_n}{N_n} \right] + [1 - D] L_n \]

\[ R_{n+1} = \left[ \frac{b_R (1 - \sigma_{RR}) R^2_n + (b_R + b_L) (1 - \sigma_{RL}) R \sigma_{LL} L^2_n}{N_n} \right] + [1 - D] R_n . \]

Note: The parameters \( b_R, b_L, \sigma_{RR}, \sigma_{RL}, \) and \( \sigma_{LL} \) are frequency dependent, but this dependence is suppressed in the above equation for the sake of clarity.

**Continuous Model**

The iterative perspective is intuitive but has limited predictive capacity. One limitation is that the number of left- and right-handers are only defined at fixed intervals. To remove this obstacle, we transform the discrete model into a continuous model. We set \( \beta_X(l) = \frac{dX(l)}{dt} \) to be the instantaneous birth rate for individuals with handedness \( X \) and \( \delta = \frac{D}{\Delta t} \) to be the instantaneous death rate. We set \( X(t) = X_n \) and \( X(t + \Delta t) = X_{n+1} \), and let \( \Delta t \to 0 \) to obtain ordinary differential equations for the evolution of \( R(t), L(t) \) and \( N(t) \). However, these equations are not independent. In fact, we are interested only in the the evolution of \( l(t) \), which is governed by

\[ \frac{dl(t)}{dt} = [\beta_R \sigma_{RR} r(t)^2 + (\beta_R + \beta_L) \sigma_{RL} r(t) l(t) + \beta_L \sigma_{LL} l(t)^2] - \beta_{eff} l(t) , \quad (S1) \]

where \( \beta_{eff} \) represents a weighted average \( \beta_{eff} = r(t) \beta_R + l(t) \beta_L \).

In order to analyse this ODE, assumptions about the functions \( \beta_X(l) \) are needed: a first order assumption is that these should be linear functions of the frequency \( l \), and by symmetry, we expect that \( \beta_L(l(t)) = \beta_R(1 - l(t)) \). In other words, the birth rates must satisfy

\[ \beta_L(l) = \beta_0 + \beta_1 (l - 1/2) \]

\[ \beta_R(l) = \beta_0 - \beta_1 (l - 1/2) . \]

where \( \beta_0 \) and \( \beta_1 \) are unknown parameters. To obtain rough estimates for the values of these parameters we note that according to Aggleton et al. [21], the average lifespan of right-handers is 3.31% longer than their left-handed counterparts, with much of the difference attributable to higher rates of premature death in war and accidents. This indicates that in a society consisting of approximately 90% right-handers, left-handers appear to have a lower fitness. This fitness differential should be reflected in the model’s reproductive rates. This suggests that at equilibrium \( (l = l^*), \beta_R(l^*) = 1.0331 \cdot \beta_L (l^*) \). We also note that census data suggests that \( \beta_{eff}(l^*) = 0.01383 [25] \). This allows us to solve for \( \beta_0 \) and \( \beta_1 \) in terms of \( l^* \). From [7], the observed fractions of left-handed offspring \( \sigma_{XY} \) are

\[ \sigma_{RR}(l^*) = 0.095 , \]

\[ \sigma_{RL}(l^*) = 0.195 , \]

\[ \sigma_{LL}(l^*) = 0.261 . \]

We expect these parameters to be functions of \( l \), but all data is drawn from modern societies where the fraction left-handed is \( l = l^* \). Fortunately, using symmetry arguments, we can obtain additional points:

\[ \sigma_{RR}(1 - l^*) = 1 - 0.261 , \]

\[ \sigma_{RL}(1 - l^*) = 1 - 0.195 , \]

\[ \sigma_{LL}(1 - l^*) = 1 - 0.095 . \]
In a population of uniform handedness, one might expect all offspring to inherit the same handedness as their parents. However, in practice the situation is more complex. Monozygotic (identical) twins often possess discordant handedness [7]. Thus, handedness cannot be fully determined by genotype. To account for this, most genetic models introduce a random component that partially determines handedness. With that motivation, we define $\epsilon_{XY}$ to be the probability due to chance that parents with handedness $XY$ produce left-handed offspring in a population consisting entirely of right-handers. We then obtain:

$$\sigma_{RR}(0) = \epsilon_{RR}$$
$$\sigma_{RL}(0) = \epsilon_{RL}$$
$$\sigma_{LL}(0) = \epsilon_{LL}$$
$$\sigma_{RR}(1) = 1 - \epsilon_{LL}$$
$$\sigma_{RL}(1) = 1 - \epsilon_{RL}$$
$$\sigma_{LL}(1) = 1 - \epsilon_{RR}.$$  

It is unclear exactly what values are appropriate for $\epsilon_{XY}$ since no isolated human population consisting entirely of left- or right-handers exists. If $\epsilon_{XY} = 0$, then equilibria at $l = 0, l^*, 1/2, 1 - l^*, 1$ appear and those at $l = l^*$ and $l = 1 - l^*$ are unstable. This is inconsistent with the observed stable fixed point. We therefore assume $\epsilon_{XY}$ must satisfy $0 < \epsilon_{XY} < \sigma_{XY}(l^*)$.

For given $\epsilon_{XY}$ values, we can fit a cubic polynomial to the 4 known points $\sigma_{XY}(l)$ (known at $l = 0, l^*, 1 - l^*, 1$) to obtain smooth approximate functions $\sigma_{XY}(l)$. The resulting dynamical system governed by equation (S1) has either 3 or 5 fixed points: $(l^*, \frac{1}{2}, 1 - l^*)$ or $(l^*, l_2, \frac{1}{2}, 1 - l_2, 1 - l^*)$ where $l^*$ and $1 - l^*$ are stable. For example, if we set $\epsilon_{RR} = \epsilon_{RL} = \epsilon_{LL} = 0.02$, we see an unstable fixed point at $l = 1/2$ in addition to the expected stable fixed points at $l = l^*$ and $l = 1 - l^*$.

Up to this point, we have treated $l^*$ as an unknown parameter. In practice, however, the fraction of the population that is left-handed can be measured. Estimates for the value of this parameter depend on the precise method of measurement and definition of left-handedness, but it is generally agreed that this fraction is close to 10% [2]. Equation (S1) allows us to compute an independent prediction for $l^*$ using only the birth rates and the phenotype ratios of offspring given above (note: the predicted $l^*$ does not depend on the choice of $\epsilon$ although the stability of the fixed point does). This results in a predicted percent left-handed of 11.78%, consistent with the measured value.

By presenting this model of phenotype evolution, we wish to emphasize the generality of the probabilistic model presented in the main text. For appropriate choices of functions $P_{RL}$, equation (2.2) in the main text can be made to agree nearly identically with model S1 above, as demonstrated in figure S2.

**Figure S2.** Comparison of probabilistic and phenotypic models. Solid blue line: the function $dl/dt$ [%/yr] generated using generic sigmoid $P_{RL}$ in the probabilistic model from equation (2.2) of the main text. Dashed red line: the function $dl/dt$ [%/yr] implied by the phenotypic model from equation (S1).
S3. THE SYMMETRY OF HANDEDNESS

There have been various attempts to account for species-level hand preferences. Billiard et al. [17] point out that left-handedness is “associated with several fitness costs.” Thus the population-level bias can be maintained through a balance between a frequency-dependent fitness function and a constant fitness cost. An alternate model by Ghirlanda et al. [10] suggests that the combination of “antagonistic” and “synergistic” interactions and their associated frequency-dependent fitness functions can create an evolutionarily stable equilibrium with an asymmetric (and non-trivial) distribution of handedness.

While both models have merit, they disagree on a fundamental question: Are left- and right-handedness interchangeable? In other words, is a mirror image world of 90% left-handers and 10% right-handers equally plausible? Billiard et al. suggest that the fitness costs “such as lower height and reduced longevity...are not likely to be frequency-dependent.” Thus these fitness costs break the symmetry and guarantee that only the observed handedness distribution is possible.

However, the aforementioned fitness costs have only been observed in a biased population consisting of lateralized individuals. For example, Aggleton et al. [21] show that left-handers are more likely to die prematurely, and that this effect is at least partially due to “increased vulnerability to both accidental death and death during warfare.” They go on to argue that “the most likely explanation for the increase in accidental death among the left-handed men concerns their need to cope in a world full of right-handed tools, machines, and instruments.” Clearly if left-handedness were more prevalent than right-handedness, then left-handed tools would also be more common, and, as a result, right-handers instead would experience increased risk of accidental death. Thus, it is likely that these fitness costs are frequency-dependent and symmetric.

In contrast to Billiard et al., Ghirlanda’s model assumes that left- and right-handedness are indeed interchangeable. Given an initial distribution of 50% left-handers and 50% right-handers, this model predicts that both the observed distribution and its mirror image are equally likely equilibrium outcomes. On this point, our probabilistic model agrees with Ghirlanda et al. Given the fact that there is no reason to expect that right-handedness is inherently superior to left-handedness from a fitness perspective, we assume that the probabilistic transition rates satisfy the symmetry condition $P_{RL}(l) = P_{LR}(1-l)$.

S4. DERIVATION OF ATHLETIC SELECTION MODEL

Professional sports are artificial systems that involve varying degrees of competitive and cooperative activities. Their participants undergo a selection process through tryouts that is in some ways analogous to natural selection. Additionally, handedness data for professional athletes is widely available. Thus, athletics provide an ideal opportunity to test whether our model’s predictions are consistent with data from selective systems.

There is a fundamental difference between selection in professional sports and natural selection. In natural selection, the distribution of a trait within a population changes in response to selection pressure, modifying the gene pool. Professional athletes, however, represent only a small segment of the much larger human population. Changes in the distribution of a trait among professional athletes are unlikely to influence the gene pool in the human population; furthermore, most professional sports have not existed for the time scales required to significantly modify the gene pool. As a result, the population of professional athletes must draw new members from a pool that consists of about 90% right-handers. Thus, direct comparison of our model to sports data is not possible: instead, we must account for this more complex selection process in order to make predictions that are applicable to real-world sports.

To begin, we define $l^*$ to be the fixed point predicted by the probabilistic model (2.2) described in the main text. This represents the ideal equilibrium distribution of left-handedness in a hypothetical world where all interaction occurs through the sport under consideration. We assume that skill is normally distributed throughout the population with mean $\mu = 0$ and standard deviation $\sigma$. Because left-handedness is relatively rare, this trait should provide a competitive advantage in sports involving direct physical confrontation. Let $l$ represent the fraction left-handed within a sport. When $l$ deviates from $l^*$, the sport is not at its ideal equilibrium state, and left-handers must experience a shift in skill $\Delta s$. We assume that professional sports operate efficiently, that is, they select players exclusively according to skill level. Then the distributions of skill among left- and right-handers satisfy

$$p_R(s) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$p_L(s) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\Delta s)^2}$$
where
\[
\Delta s = k (l^* - l).
\]

In this formulation, when the ideal equilibrium fraction of left-handedness is achieved, their is no advantage to possessing either handedness and the individuals are selected according their intrinsic skill.

In most sports, individuals must undergo a tryout in order to demonstrate sufficient skill for participation. As a result only a fraction of the total population is allowed to participate in the sport at a given level of competition. We define the fraction selected \( \psi = n/N \), where \( n \) is the number of individuals selected and \( N \) is the size of the total population. \( \psi \) will determine a minimum skill cutoff \( s_c \) for participation according to the relation 
\[
\psi = l_{bg} \int_{s_c}^{\infty} p_L (s) \, ds + (1 - l_{bg}) \int_{s_c}^{\infty} p_R (s) \, ds,
\]
where \( l_{bg} \) represents the background rate of left-handedness (\( \approx 1/10 \)). We can simplify this expression by normalizing the various parameters by the standard deviation: we set \( \hat{s}_c \equiv s_c/\sqrt{2\sigma^2} \), \( \hat{\Delta s} \equiv \Delta s/\sqrt{2\sigma^2} \), and \( \hat{k} \equiv k/\sqrt{2\sigma^2} \) to get
\[
\psi = \frac{1}{2} l_{bg} \text{erfc} (\hat{s}_c - \hat{\Delta s}) + \frac{1}{2} (1 - l_{bg}) \text{erfc} (\hat{s}_c).
\] (S2)

In this formulation, the fraction left-handed among the individuals selected will be represented by the first term on the right-hand side of equation (S2) divided by the entire expression,
\[
l_{sel} = \frac{l_{bg}}{2\psi} \text{erfc} (\hat{s}_c - \hat{\Delta s}).
\]

After many tryouts, the fraction left-handed within a sport will stabilize with the \( l_{sel} = l \). This equilibrium fraction represents the observed fraction left-handed among professional athletes, thus we call it \( l_{pro} \). So, the equilibrium state is implicitly determined by
\[
l_{pro} = \frac{l_{bg}}{2\psi} \text{erfc} \left[ \hat{s}_c - \hat{k} (l^* - l_{pro}) \right].
\] (S3)

This model has the following properties:

- If \( l^* = l_{bg} \), then \( l_{pro} = l_{bg} \).
- If the sport is not selective at all, in the limit \( s_c \to -\infty (\psi \to 1) \), \( l_{pro} \to l_{bg} \) as this means that selection is independent of skill.
- As the sport becomes infinitely selective, \( s_c \to \infty (\psi \to 0) \), \( l_{pro} \to l^* \). In other words, the ideal equilibrium fraction is achieved when the sport is infinitely selective. (To see this, assume that \( l_{sel} = l^* + \delta \) and \( l = l^* - \delta \). Expand in a Taylor series about \( \delta = 0 \) and take the limit \( s_c \to \infty \) to find that \( \delta \to 0 \). Thus \( l_{sel} \to l^* \) and \( l \to l^* \), so \( l_{pro} = l^* \).)

Using this model, we employed numerical techniques to compute the solutions \( l_{pro} \) for a variety of sports, and then compared these results to the observed fractions left-handed as seen in figure 2.

This model can also be extended to examine how the distribution of handedness varies within a single sport. If left-handedness is a desirable trait (that is, it provides a skill advantage at equilibrium), then we expect that it should be very prevalent among the most skilled individuals due to the selection mechanism. To see this, we consider the case of baseball. It is clear that in baseball, \( l^* = \frac{1}{2} \) since the sport involves primarily competitive interactions (the observed \( l_{pro} \approx 0.3 \)). Thus, left-handedness is a desirable trait for potential professionals as it will provide a particular skill advantage in batting. At the professional level, most hitters face the same set of pitchers and compete indirectly with one-another for roster spots. They should therefore be expected to experience the same skill advantage due to handedness. In other words, \( \Delta \hat{s} = \hat{k} (l^* - l_{pro}) \) is a constant (in sports like boxing, however, where individuals compete more frequently with others near their own rank, the skill advantage would be a rank-dependent function \( r \), \( \Delta \hat{s}_r = \hat{k} (l^* - l_r) \)). Ranking the hitters by skill, we observe that the fraction left-handed above rank \( r \) should satisfy
\[
l_r = \frac{l_{bg} N}{2r} \text{erfc} (\hat{s}_r - \hat{\Delta s}),
\]
where \( s_r \) satisfies

\[
    r = \frac{1}{2} h_{bg} \text{erfc} (s_r - \Delta s) + \frac{1}{2} (1 - h_{bg}) \text{erfc} (s_r) .
\]

Using this result, we plotted the predicted fraction left-handed as a function of rank as seen in figure 3. This model predicts a non-trivial shape for the distribution of handedness within baseball that is consistent with the observed distribution. This is a strong indication that this athletic selection model provides a good mathematical approximation for the tryout-based selective mechanism within professional sports.

S5. DATA SUMMARY

Data used in generation figure 2 came from a variety of sources. The total number of participants came from surveys conducted by the Sporting Goods Manufacturers Association in 2009 [26], except for men’s and women’s fencing, where participant numbers were extrapolated from data published by the National Federation of State High School Associations [27]. The number of professional players came from listings of top-rated players (the only ones for which handedness was readily available) at the internet URLs indicated in Table S1, with the exception of baseball, football, and hockey, where numbers are absolute totals.

When handedness was not available in tabulated form, it was evaluated based on public photos of players in action.

In Table S1, the predictions for the fraction left-handed were generated using an estimate of the ideal equilibrium \( l^* \) for each sport. The appropriate value for \( l^* \) depends primarily on the degree of cooperation \( c \) for the sport. This parameter is difficult to estimate in sports that possess clear cooperative and competitive elements. However, in order to observe fixed points other than \( l^* = 1/2 \), \( c \) must exceed a threshold that appears to be relatively high for the types of transition rates considered in this paper (See figure 1). So, we assumed that \( l^* = 1/2 \) for sports primarily involving direct confrontations: baseball (batters vs. pitchers), boxing, fencing, table tennis, hockey (defensemen and forwards).

Some sports (or particular positions within sports), however, possess highly lateralized equipment, positioning or strategy. For these sports, it is ideal for all individuals to possess the same handedness; so, the minority handedness will be selected against. For example, in football, blocking schemes are often designed to protect a quarterback’s blind side. As a result, it is beneficial for all quarterbacks on the roster to possess the same handedness in order to minimize variations of the offensive sets. Consequently, we assume that for quarterbacks in football, golfers, and left and right wings in hockey the value of \( c \approx 1 \), i.e., \( l^* = 0 \) or \( 1 \).

S6. PARAMETER SENSITIVITY ANALYSIS FOR PROBABILISTIC MODEL

In the probabilistic model, there are two unknown functions \( P_{RL}^{\text{coop}}(l) \) and \( P_{RL}^{\text{comp}}(l) \). While general properties of these functions such as monotonicity are known, the appropriate form for these functions is unknown and is difficult to determine from data. The generic sigmoid functions

\[
P_{RL}^{\text{comp}}(l) = \left[ 1 + e^{k_1 \frac{l - l^*}{c}} \right]^{-1}, \quad \text{(S4a)}
\]

\[
P_{RL}^{\text{coop}}(l) = \left[ 1 + e^{-k_2 \frac{l^* - l}{c}} \right]^{-1}, \quad \text{(S4b)}
\]

satisfy restrictions on \( P_{RL} \) for \( k_1, k_2 > \sqrt{3}/2 \) and capture the essential fixed point behaviour (\( k_1, k_2 \) set the steepness of the curves). Unfortunately, these equations introduce two new parameters that may alter the dynamics. To examine the sensitivity of the model to these parameters, we assumed \( l^* \) was the fixed point of the system. We then computed the partial derivatives of \( l^* \) with respect to each parameter. In the vicinity of \( l^* = 1/10 \), the observed ratio of left-handers in human populations, and in the range of \( k_1 \in \left[ \frac{\sqrt{3}}{2}, \infty \right) \), \( k_2 \in \left[ \frac{\sqrt{3}}{2}, \infty \right) \) and \( c \in (0, 1) \), we found that

\[
    \left| \frac{\partial l^*}{\partial c} \right| \geq O(10^2) \quad \text{and} \quad \left| \frac{\partial l^*}{\partial k_1} \right| \geq O(10^2) .
\]

Thus for a wide range of parameter values, \( \left| \frac{\partial l^*}{\partial c} \right| \gg \left| \frac{\partial l^*}{\partial k_1} \right| \) and \( \left| \frac{\partial l^*}{\partial k_2} \right| \gg \left| \frac{\partial l^*}{\partial k_1} \right| \) near the fixed point \( l^* = \frac{1}{10} \). In other words, the location of the fixed point is more sensitive to \( c \) than \( k_1 \) and \( k_2 \) by several orders of magnitude.
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<td>Federation Internationale D’Escrime, top 25 ranked for each weapon: Epee, Saber and Foil, 2010</td>
<td>1462</td>
<td>75</td>
<td>0.200</td>
<td>0.247</td>
<td>Handedness</td>
<td>fie.ch</td>
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<td>Fencing, Women</td>
<td>Federation Internationale D’Escrime, top 25 ranked for each weapon: Epee, Saber and Foil, 2010</td>
<td>1123</td>
<td>75</td>
<td>0.253</td>
<td>0.240</td>
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<td>fie.ch</td>
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<td>Football, Quarterbacks</td>
<td>National Football League, active players, preseason 2010</td>
<td>75071</td>
<td>120</td>
<td>0.067</td>
<td>0.064</td>
<td>Google image</td>
<td>nfl.com</td>
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<td>pgatour.com</td>
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<td>1000000</td>
<td>100</td>
<td>0.000</td>
<td>0.060</td>
<td>LPGA Tour profile images</td>
<td>lpga.com</td>
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<tr>
<td>Hockey, Defensemen and Forwards</td>
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<td>309971</td>
<td>601</td>
<td>0.323</td>
<td>0.303</td>
<td>Shot side</td>
<td>espn.go.com/nhl</td>
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<tr>
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<td>80458</td>
<td>156</td>
<td>0.090</td>
<td>0.065</td>
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<td>0.613</td>
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<tr>
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<td>International Table Tennis Federation, top ranked players, November 2010</td>
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<td>Ranking photo</td>
<td>ittf.com</td>
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<td>Table Tennis, Women</td>
<td>International Table Tennis Federation, top ranked players, November 2010</td>
<td>654372</td>
<td>100</td>
<td>0.200</td>
<td>0.327</td>
<td>Ranking photo</td>
<td>ittf.com</td>
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Table S1. Summary of data sources used in preparing figure 2.
for physically allowable $k$-values. Therefore we are justified in ignoring the effects of individual choices of $k_1$ and $k_2$ in order to focus on the effects of the choice of $c$. We believe these results are robust for various different sigmoid functions $P_{RL}^{comp}, P_{RL}^{coop}$.

Note: See manuscript for references 1-24.